



# Linear Algebraic Modeling of Power Flow in the HMPT500-3 Transmission

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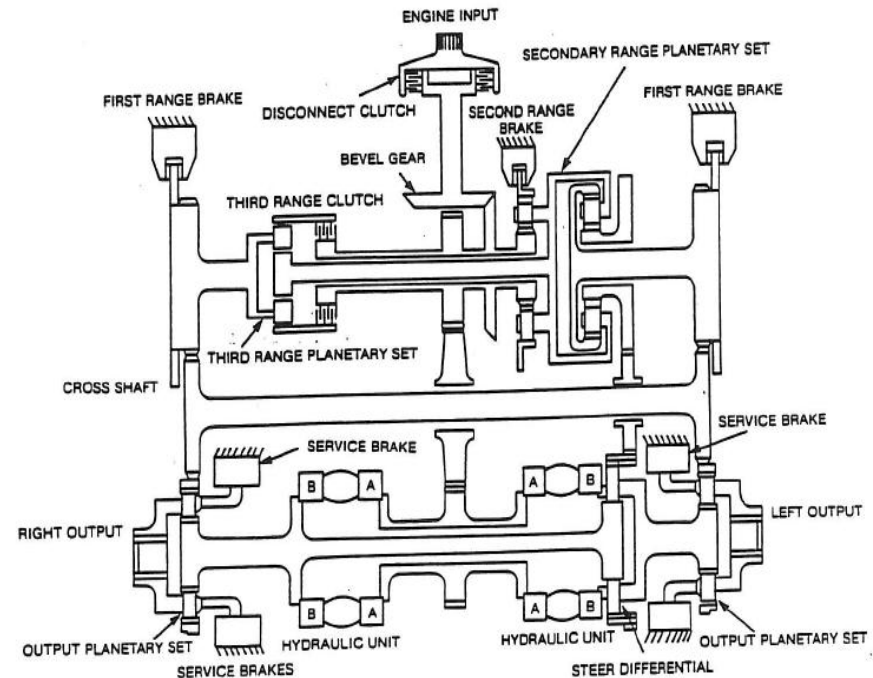
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# HMPT500-3 Introduction

# MSTV

MODELING AND SIMULATION, TESTING AND VALIDATION

- Tracked Vehicle Cross-Drive Transmission
  - Split-Torque Path Hydrostatic / Mechanical CVT
  - Six Planetary Gear Sets
    - Three in “Range Pack”
    - Two in outputs (L / R)
    - One double-planet Steer Differential
  - Two Hydrostatic Pump / Motor Units (HSU)
    - “A” end Variable Displacement controlled by actuator stroke
    - “B” end fixed Displacement





- Discreet Mechanical Ratio selected in Range Pack
  - 1<sup>st</sup> & 2<sup>nd</sup> Range Brakes
  - 3<sup>rd</sup> Range Clutch
- HSU A-end displacements changed equally to continuously add / subtract speed from output planetaries
  - $-0.85 < Z_{L,R} < +0.85$ , remainder of stroke reserved for steer
- HSU L / R A-end displacement changed differently to provide steering
  - Steer stroke difference superimposed on ratio
    - $dZ_L + dZ_R = 0$  to maintain forward speed
    - $-1.0 < Z \pm dZ_{L,R} < +1.0$



# HMPT500-3 Simplified Model

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- Simplified treatment for forward operation (no steering)
  - L / R HSUs combined into single unit
  - L / R Output Planetaries combined
  - Steer Differential becomes a fixed ratio
- Model has Four Planetaries, One HSU
  - Each Planetary has sun gear, planet carrier, ring gear elements
  - HSU has A-end, B-end elements
  - Total 14 elements requiring 14 independent equations for speed and torque
- Models ideal (lossless) power flow
  - Throughput & spin losses estimated after solution

# Planetary Gear Equations



- General formulation for planetary differential (all elements turning)
  - Clutch / Brake application handled in constraint equations
- Sign convention for speed & torque +ve for power input to element
- One planetary speed equation
  - $$\frac{N_1}{(N_1 + N_3)} * S_1 - S_2 + \frac{N_3}{(N_1 + N_3)} * S_3 = 0 \quad (1)$$
- Two planetary torque equations
  - $$\frac{1}{N_1} * T_1 + \frac{2}{(N_1 + N_3)} * T_2 + \frac{1}{N_3} * T_3 = 0 \quad (2)$$
  - $$\frac{1}{N_1} * T_1 - \frac{1}{N_3} * T_3 = 0 \quad (3)$$

# HSU Equations



- HSU treated as variable speed-ratio producer
  - Ratio directly proportional to stroke  $Z$
  - Ignores hydraulic leakage
- One HSU speed equation
  - $Z * S_A - S_B = 0; -1 \leq Z \leq 1 \quad (4)$
- One HSU torque equation
  - $T_A + Z * T_B = 0 \quad (5)$



# Intermediate Gear Equations



- Define intermediate gear / shaft connections between planetary & HSU elements
  - Gear ratios implicit in equations
- One speed equation for each *pair* of elements with defined speed ratio
  - $S_m - R_{m,n} * S_n = 0 \quad (6)$
- One torque balance for each *set* of coupled elements, eg cross shaft
  - $\sum \frac{T_m}{R_{m,n}} = 0 \quad (7)$



# Clutch / Brake Equations



- Modified constraints for each forward range (1<sup>st</sup> – 3<sup>rd</sup>) based on which clutches, brakes applied

	Clutch	Brake
Released	Speed = calculated <sup>(1)</sup> Torque = 0	Speed = calculated Torque = 0 <sup>(2)</sup>
Engaged	Speed = imposed Torque = calculated <sup>(3)</sup>	Speed = 0 Torque = calculated
<p>(1) Speed calculated from planetary equation (2) <math>T_{\text{brake}} = 0</math>, other torque balance may apply (3) Torque calculated from planetary &amp; torque balance eq.</p>		



# External Boundary Conditions



- Engine input speed & torque applied to input bevel shaft
- Only non-homogeneous constraints
  - Speed imposed through input bevel ratio on:
    - 2<sup>nd</sup> Range Input Stage Sun Gear
    - 3<sup>rd</sup> Range Ring Gear (if clutch engaged)
    - HSU A-end through intermediate ratio
  - Torque applied to input bevel shaft torque-balance, including above elements

# System of Linear Equations



- Two independent 14x14 matrix equations
  - Speed:  $[a] * \{S\} = \{b\}$  (8)
  - Torque:  $[c] * \{T\} = \{d\}$  (9)
- Ideal (lossless) power flow calculated from speed / torque solution vectors
  - $P_i = \frac{T_i * S_i}{5252}$  (10)
- Losses deducted from ideal power flow
  - Spin losses
    - Input driven makeup / auxiliary pumps
    - HSU no-load spin losses
  - Throughput loss factors (per gear mesh, HSU element)

$$\{P_{loss}\} = \begin{bmatrix} f_1 & & \\ & f_2 & \\ & & f_n \end{bmatrix} * \{P_{ideal}\} \quad (11)$$

# Model Results, Overview

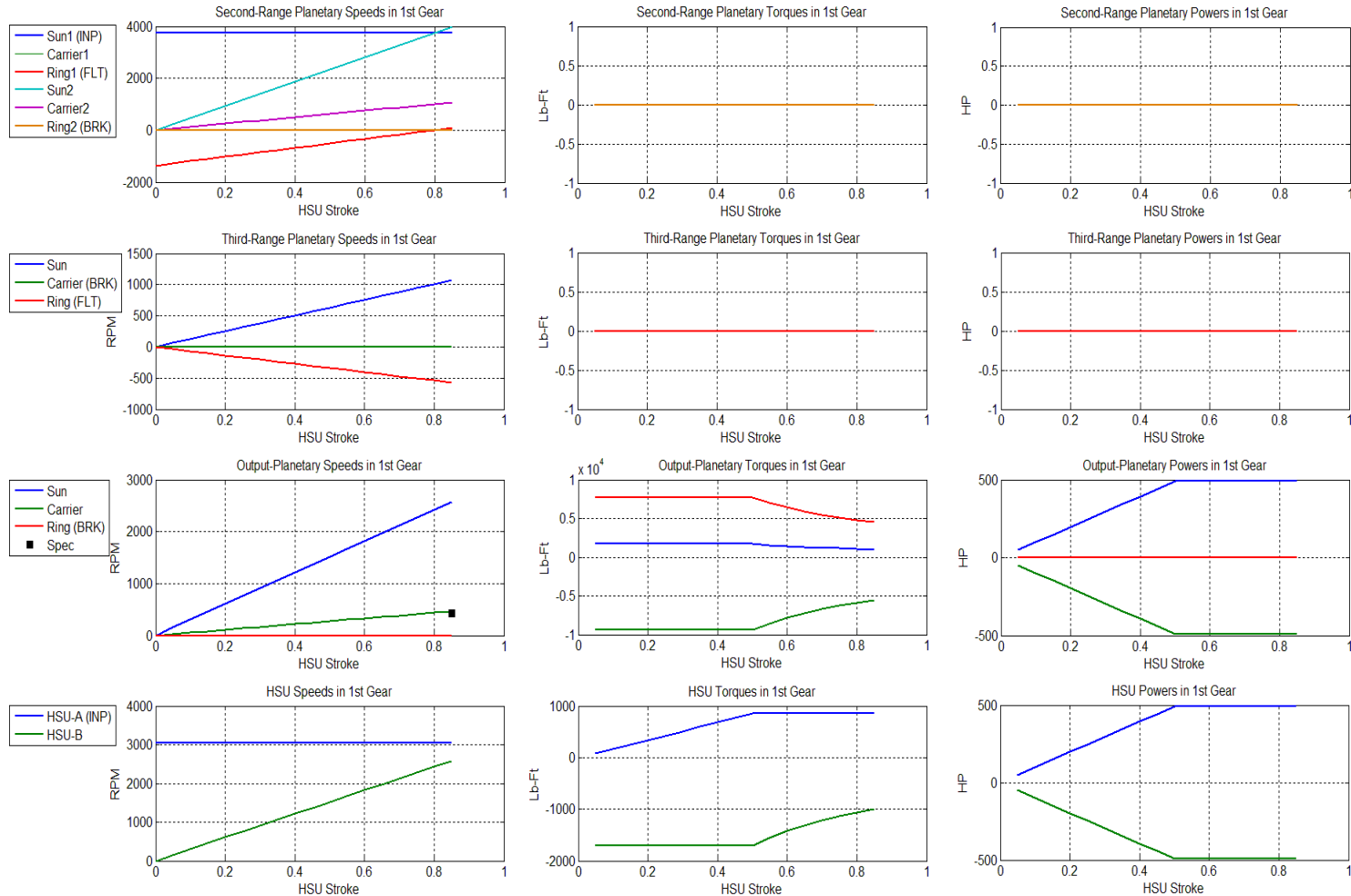


- Engine Speed 2600 rpm
- Engine Torque 1010 lb-ft (500HP after fan PTO)
- 1<sup>st</sup> Range
  - Stroke  $0 \leq Z \leq +.85$
  - Torque ramps up from zero as Z increases
  - 1<sup>st</sup> Range brakes applied
- 2<sup>nd</sup> Range
  - Stroke  $+.85 \geq Z \geq -.85$  (*note direction*)
  - 2<sup>nd</sup> Range brake applied
- 3<sup>rd</sup> Range
  - Stroke  $-.85 \leq Z \leq +.85$
  - 3<sup>rd</sup> Range clutch engaged

# 1<sup>st</sup> Range Results

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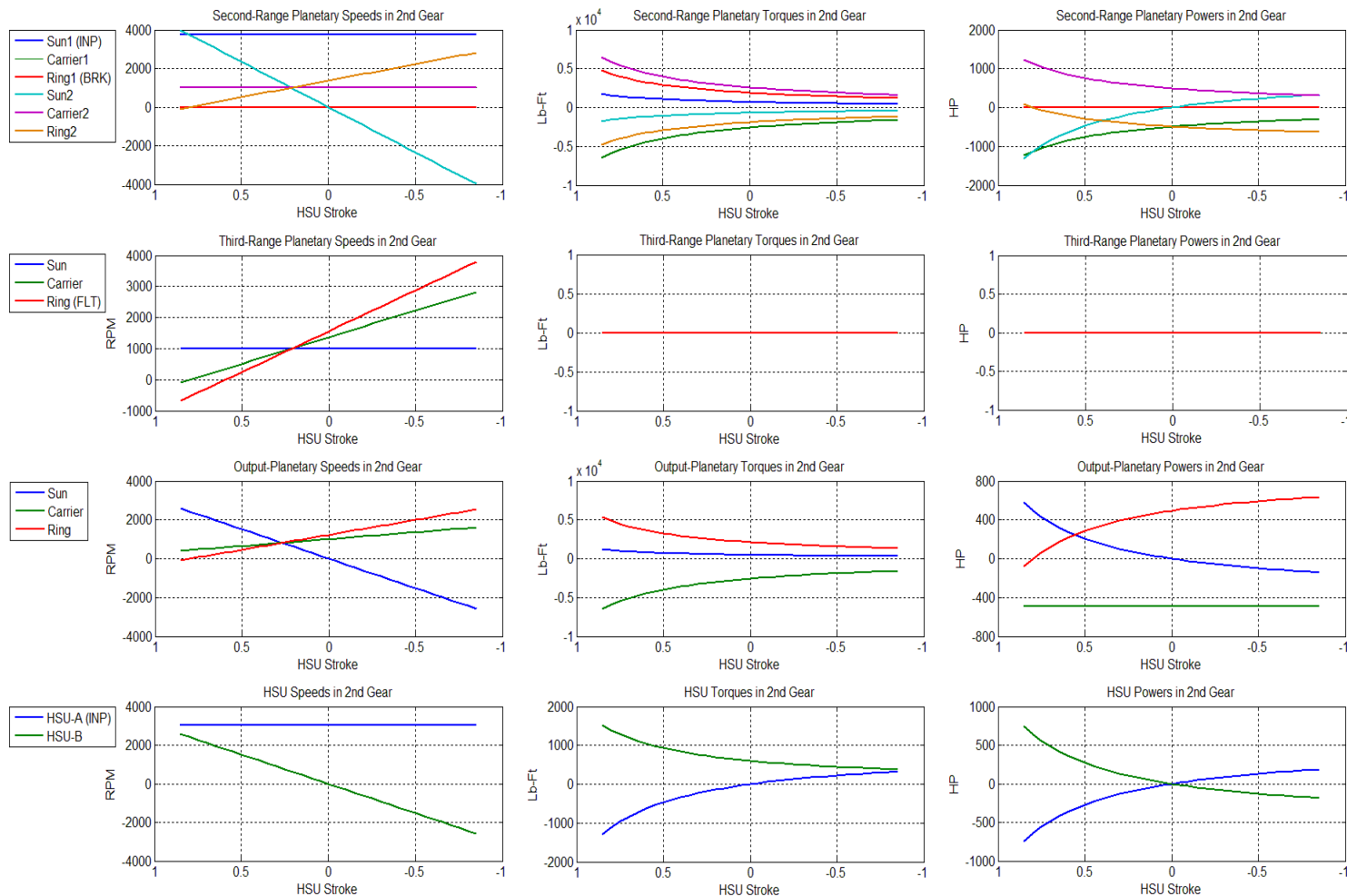
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# 2<sup>nd</sup> Range Results

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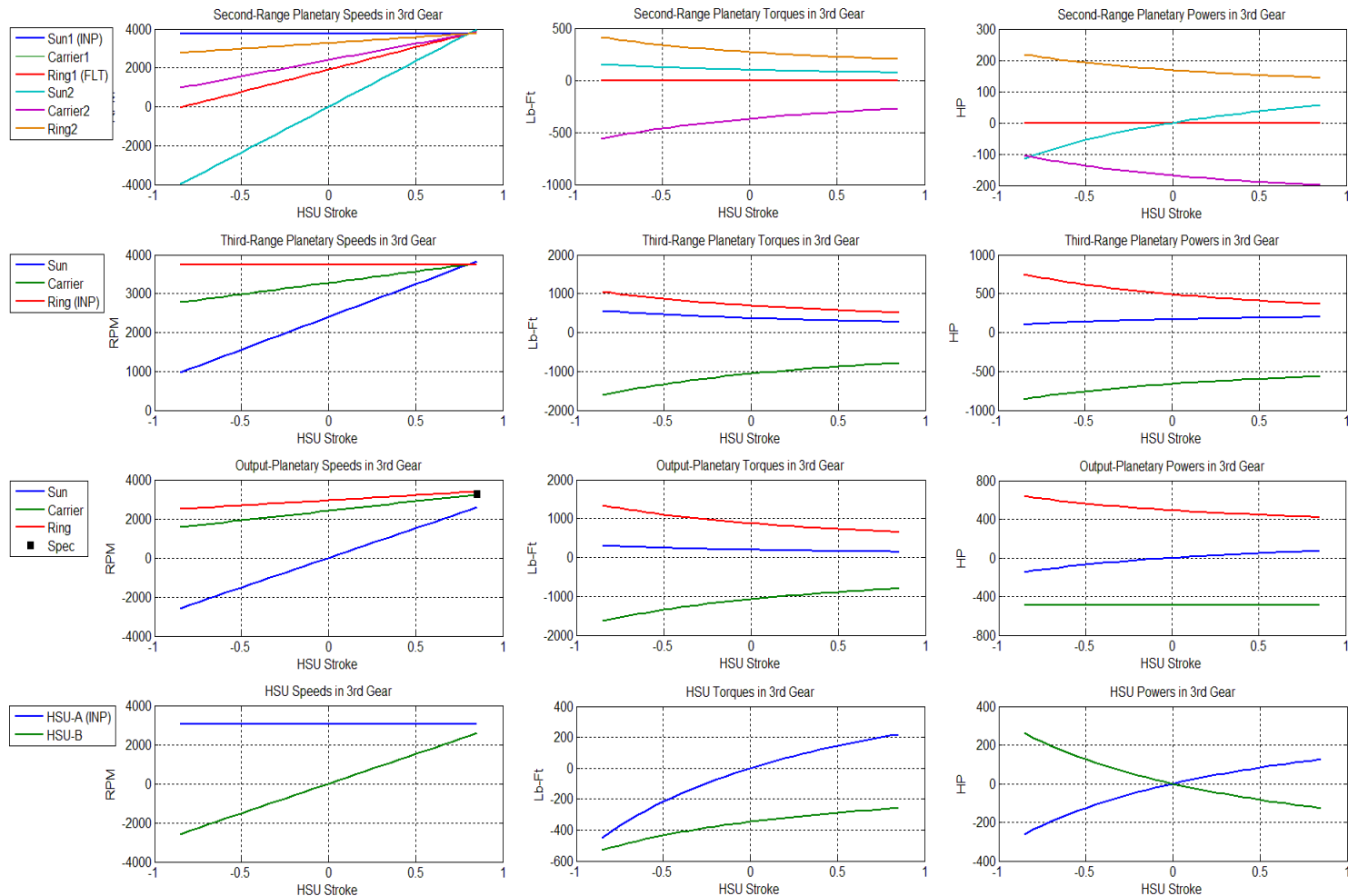
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# 3<sup>rd</sup> Range Results

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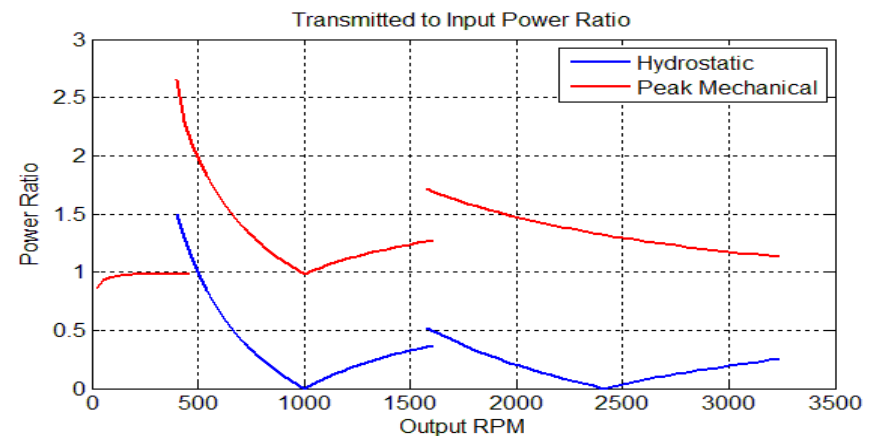
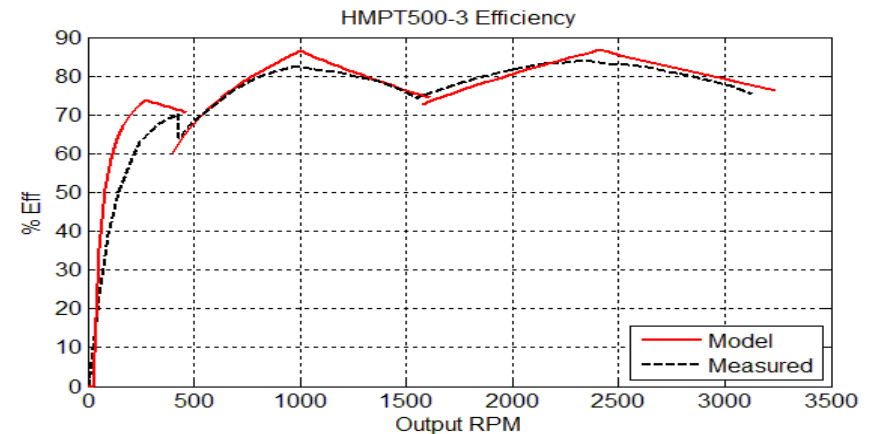
# Transmission Efficiency

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- Deduct losses from transmission outputs
  - Spin Losses
    - Makeup / Auxiliary Pump
    - HSU spin losses
  - Throughput Losses
    - 0.5% per gear mesh
    - 5% per HSU element (“A”, “B” end)
- Calculate Efficiency
  - Good agreement with test data
  - Correlates to Power Ratios
    - Hydrostatic
    - Reactive Power





- Simplified Linear Model
  - Reproduces HMPT500-3 kinematics for forward operation
  - Conserves power / energy overall
  - Enables estimation of transmission losses, efficiency
- Expanded Linear Model
  - 6 planetary gears, 2 HSUs: 22 simultaneous equations speed / torque
  - Model both steering and drive ratio